6 $\log z = \log 3 + \frac{z-3}{3} - \frac{(z-2)^2}{\lfloor 2\, 3^2} + \cdots$

$+ \frac{(z-3)^3}{\lfloor 3\, 3^3} + \cdots$ where $|z-3| < 1$

P.G. sem-2nd
paper-VI, unit-1st
Complex integration
Dr. G.D. singh.

टिप्पणी .....

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सितम्बर 2009

9 बुध

Let $f(z) = \log z$

By Taylor's theorem

$$f(z) = \sum_{n=0}^{\infty} \frac{(z-a)^n f^n(a)}{\lfloor n}$$

$$= f(a) + (z-a) f'(a) + \frac{(z-a)^2}{\lfloor 2} f''(a) + \cdots \quad (1)$$

Let $a = 3$, we get -

$f(3) = \log 3$, $f'(z) = \frac{1}{z} = z^{-1}$

$f''(z) = -1z^{-2}$

$f'''(z) = 2z^{-3}$

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$\therefore f(3) = \log 3$, $f'(3) = \frac{1}{3}$, $f''(3) = -\frac{1}{3^2}$

$f'''(3) = \frac{2}{3^3}$ from (1), $a = 3$

$$f(z) = f(3) + (z-3) f'(3) + \frac{(z-3)^2}{\lfloor 2} f''(3) + \frac{(z-3)^3}{\lfloor 3} f'''(3) + \cdots$$

$$\& \log z = f(3) + (z-3)\tfrac{1}{3} + \frac{(z-3)^2}{\lfloor 2} \cdot \frac{1}{3^2} + \frac{(z-3)^3}{\lfloor 3} \cdot \frac{2}{3^3} + \cdots$$

P.T.O.

रवि	सोम	मंगल	बुध	गुरु	शुक्र	शनि
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

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11 शुक्र

(7) Obtain the Laurent's Series expansion

$$\frac{\sinh z}{z^2} = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{1}{\lfloor 2n+1} z^{2n-1}$$

Let $f(z) = \sinh z$

$\therefore f(0) = \sinh 0 = 0$

$f'(z) = \cosh z$.

$f'(0) = \cosh 0 = 1$

$f''(z) = \sinh z$

$f''(0) = \sinh 0 = 0$

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$f'''(z) = \cosh z$

$f'''(0) = \cosh 0 = 0$ 1

$\therefore f'(z) = f(0) + z f'(0) + \frac{(z-0)^2}{\lfloor 2} f''(0) + \frac{(z-0)^3}{\lfloor 3} f'''(0) + \text{---}$

$\therefore \sinh z = 0 + z(1) + 0 + \frac{z^3}{\lfloor 3} + 0 + \frac{z^5}{\lfloor 5} + \cdots$

$$\frac{\sinh z}{z^2} = \frac{1}{z} + \sum_{n=1}^{\infty} \frac{1}{\lfloor 2n+1} z^{2n-1}$$

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रवि	सोम	मंगल	बुध	गुरु	शुक्र	शनि
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15

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(8.) Expand the function $\frac{1}{(z-1)(z-3)}$ 13 रवि

in Taylor's Series about $z=2$ and indicate the circle of convergence.

The distances of both the singularities $z=1$, $z=3$ from the centre $z=2$ is 1. So $f(z)$ is regular within $|z-2|=1$ which is the circle of convergence.

Also Taylor's expansion about $z=2$ is

$$f(z) = \frac{1}{(z-1)(z-3)} = \frac{1}{z^2-4z+3}$$

$$= \frac{1}{z^2-4z+4-1}$$

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$$= \frac{1}{(z-2)^2-1}$$

$$= \frac{-1}{\{1-(z-2)^2\}}$$

$$= -\{1-(z-2)^2\}^{-1}$$

$$= -\left[(z-2)^2+(z-2)^4+(z-2)^6+\cdots\right]$$

$$= -\sum_{h=1}^{\infty}(z-2)^{2n}$$

रवि	सोम	मंगल	बुध	गुरु	शुक्र	शनि
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